contain negative substitutions may be found in exactly the same manner as those of order 8. Hence there are just 16 groups of degree 10 that contain five systems of intransitivity,—1 of order 2, 4 of order 4, 6 of order 8, 4 of order 16, and 1 of order 32. All those that have the same order represent the same abstract group.

Cornell University, November, 1898.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from May Number.]

PROPOSITION XXXVI. *If any straight XF* (Fig. 44) *makes an acute angle with any ordinate LF* [*of the equisdistantial*], *the point X does not fall without the cavity of the curve, unless previously XF has cut the curve in some point O.*

PROOF. It is certain that the point X may be assumed in XF so near to the point F, that the join LX previously cuts the curve in some point S: otherwise XF either does not fall wholly without the cavity of the curve, and so we have our assertion; or so it does not make with FL an acute angle, rather it would be to suppose that XL comes together with LF in one same straight.

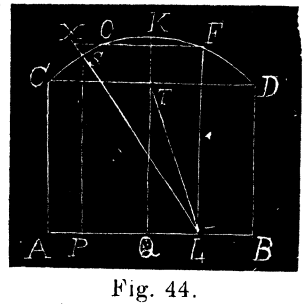

Fig. 44.

Accordingly from the point S let fall to the base AB the perpendicular SP. This will be (from P. 34) equal to LF.

But SP is (from Eu. I, 18) less than LS.

Therefore also LF is less than LS, and therefore much less than LX. Hence in triangle LXF the angle at the point X will be acute, because less (from Eu. I, 18) than the angle LFX supposed acute.

Now let fall to FX the perpendicular LT. This falls (because of Eu. I, 17) toward the parts of each acute angle. Wherefore the point T will lie between the points X and F. Then from the point T let fall to the base AB the perpendicular TQ.

LF (because of the right angle at T) will be greater than LT, and this (because of the right angle at Q) will be greater than QT. Therefore LF will be far greater than QT. But hence, if in QT produced QK is taken equal to LF, the point K (from P. 34) will pertain to the present curve, and therefore the point T falls within the cavity of this curve.

Therefore the straight FT, which cuts two straights QK and LT in T, can

not attain to cutting LS produced in the point X, situated without the cavity of the present curve, unless previously the prolonged FT' cuts in some point O the portion of this curve between the points S and K.

Hoc antem erat demonstrandum.

COROLLARY. And hence flows manifestly, that between the tangent of this curve and the curve itself cannot be placed any straight [ray], whether on one or the other side of the tangent, which falls wholly without the cavity of the curve; since a straight [ray] so located must make (from the preceding) an acute angle with the perpendicular let fall from the point of contact to the opposite base.

[To be Continued.]

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

102. Proposed by ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Iowa.

A's age is to B's as 2:3. 20 years from now their ages will be to each other as 4:5. What are their ages, respectively?

I. Solution by J. OWEN MAHONEY, B. E., M. Sc., Master of Mathematics and Science, Carthage Graded and High School, Carthage, Texas.

It is easily seen that A's age : A's age + 20 years :: 2 : 4,

or A's age : 20 years :: 2 : 2.

$\therefore$ A's age = 20 years. But B's age = $\frac{3}{2}$ of A's = 30 years.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

In the first instance, A's age = $\frac{2}{3}$ B's age.

In the second instance, A's age = $\frac{4}{5}$ B's age.

$\therefore$ $\frac{2}{3}$ B's age + 20 years = $\frac{4}{5}$(B's age + 20 years).

$\therefore$ $\frac{2}{15}$ B's age = 4 years.

$\therefore$ B's age = 30 years, A's age = 20 years.

An algebraic solution was furnished by Charles C. Cross.

Solutions of problems 98 and 99 were received from J. K. Ellwood, and solutions of problems 100 and 101 were received from J. Scheffer too late for credit in last issue.

ALGEBRA.

89. Proposed by G. A. MILLER, Ph. D., Instructor in Mathematics, Cornell University, Ithaca, New York.

Solve by quadratics, $x^2 + y = 7$ .... (1),

$x + y^2 = 11$ ..... (2).